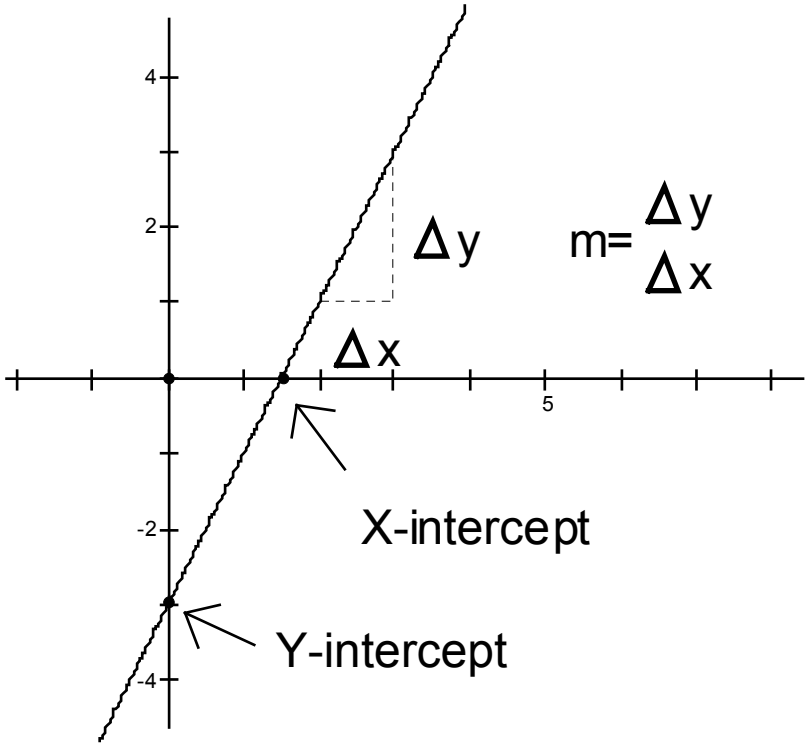
Linear Equations

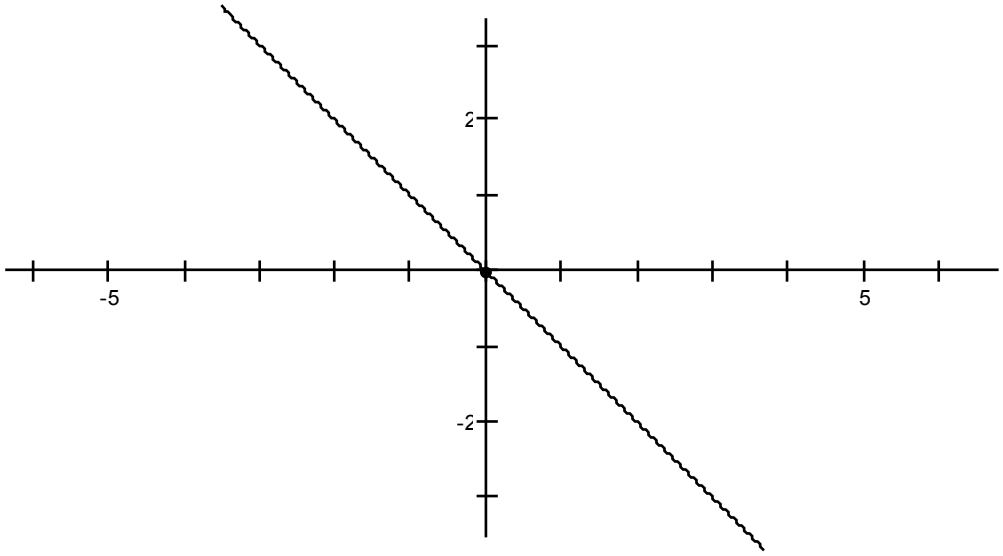
Slope Intercept Form

$y = mx + b$

In this form of a linear equation in x and y the constant $m = \frac{\triangle y}{\triangle x}$ is the slope of the line, and b is the Y intercept, the place where the line crosses the Y-axis.



Note: this line goes up from left to right so it has $m>0$.



This line goes down from left to right so $m<0$.

If $m=0$, the line has zero slope and is horizontal.

A vertical line has an undefined slope.
The equation of a vertical line will be of the form: $x = c$ where c is a constant.

Given two points, find the equation of the line that passes through them

$(2,1),(8,5)$

First find the slope

$$m = \frac{\triangle y}{\triangle x} = \frac{5-1}{8-2} = \frac{4}{6} = \frac{2}{3}$$

The equation of the line is then $y = \frac{2}{3}x + b$

To find *b* substitute either of the two points and solve for *b*

$$1 = \frac{2}{3}(2) + b$$
$$1 = \frac{4}{3} + b$$
$$b = 1 - \frac{4}{3} = -\frac{1}{3}$$

So the equation is

$$y = \frac{2}{3}x - \frac{1}{3}$$

An alternative is to use the **Point-Slope form of a line**

$$y - y_1 = m(x - x_1)$$

or

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Parallel and Perpendicular lines

Two lines with undefined slope will be parallel.

If one line has zero slope and the other has undefined slope, the two lines will be perpendicular.

Otherwise lines with the same slope will be parallel.

Perpendicular lines have **negative-inverse** slopes.

Example:

Find the equation of a line parallel to $4x+6y+5=0$ that goes through the point $(5,2)$.

Putting the equation into slope intercept form:

$y=-\frac{2}{3}x-\frac{5}{6}$ we find that

$m=-\frac{2}{3}$

So the equation of the new line is

$y=-\frac{2}{3}x+b$

Plugging in (5,2) we solve for *b*

$2=-\frac{2}{3}(5)+b$

$b=2+\frac{10}{3}=\frac{16}{3}$

So the new equation is

$y=-\frac{2}{3}x+\frac{16}{3}$

Example:

Find the equation of the line perpendicular to the line

$4x+6y+5=0$ that goes through the point $(0,0)$.

The slope of the first line is $m=-\frac{2}{3}$ so the new line which is perpendicular has a negative reciprocal slope:

$$m=-\frac{1}{\left(-\frac{2}{3}\right)}=\frac{3}{2}$$

The equation of the new line is then

$$y=\frac{3}{2}x+b$$

Plugging in (0,0) and solving for *b* we find

$$0=\frac{3}{2}(0)+b$$
$$b=0$$

So the solution is

$$y=\frac{3}{2}x$$

Point-Slope Form of the Equation of a Line

An alternate version of the equation of a line is the point/slope form

$$y - y_1 = m(x - x_1)$$

Given two points $(x_1, y_1), (x_2, y_2)$ you can plug in and find

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Example:

Find the line through points $(1,3), (4,6)$

$$y - 3 = \left(\frac{6-3}{4-1}\right)(x-1)$$
$$y - 3 = (1)(x-1)$$
$$y = x - 1 + 3 = x + 2$$
$$y = x + 2$$

Derivation of Point Slope Form

We start with an equation in standard form:

$y = mx + b$ and a point (x_1, y_1) on the line.

So we know that

$y_1 = mx_1 + b$
or
$b = y_1 - mx_1$

So we can rewrite the equation as

$y = mx + y_1 - mx_1$

Rearranging this we have

$y - y_1 = m(x - x_1)$

Finding parallel and perpendicular lines using point/slope form.

Example:

Given the line $y = \frac{2}{3}x - \frac{5}{3}$

Find a line parallel that passes through the point (2,-1)

Since the line is parallel we know that the slope is $\frac{2}{3}$

$y = \frac{2}{3}x + b$

Plugging in (2,-1) we solve for *b*

$-1 = \frac{2}{3} \cdot 2 + b$

$b = -1 - \frac{4}{3} = -\frac{7}{3}$

So we have a solution

$y = \frac{2}{3}x - \frac{7}{3}$

Now find a line perpendicular that passes through (2,-1)

Here we know the slope is the negative reciprocal $-\frac{3}{2}$

$y = -\frac{3}{2}x + b$

Plugging in (2,-1) we solve for *b*

$-1 = -\frac{3}{2} \cdot 2 + b$

$b = -1 + 3 = 2$

So we have a solution

$y = -\frac{3}{2}x + 2$

Vertical and Horizontal lines

Note the methods mentioned above may break down in the case of horizontal or vertical lines.

If the slope of a line is zero, then it is a horizontal line

The equation of a horizontal line is $y=c$ where c is a constant

If the slope of a line is undefined, then the line is vertical.

the equation of a vertical line is $x=d$ where d is a constant.

Note that all vertical lines are parallel to each other.

Also all horizontal lines are parallel to each other.

Any vertical and horizontal line are perpendicular.

Example:

What is the equation of the vertical line passing through (-3,2)?

$x=2$

What is the equation of a line parallel to

the line $y=4$ that passes through (5,7)?

Since this is a horizontal line, the solution is $y=7$.

Graphing Linear Inequalities

Graphing a linear inequality is started in the same manner as a linear equality.

The first step is usually to put the equation in slope intercept form.

Example:

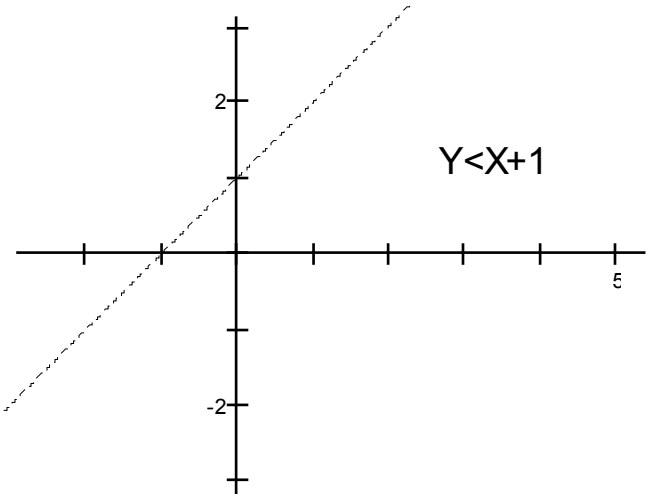
$y - x - 1 < 0$

$y < x + 1$

If the inequality is less than or equal to, or greater than or equal to, start by drawing a solid line.

If the inequality does not have "or equal to", draw the line as a dashed line to indicate that the line is not part of the solution.

Example:

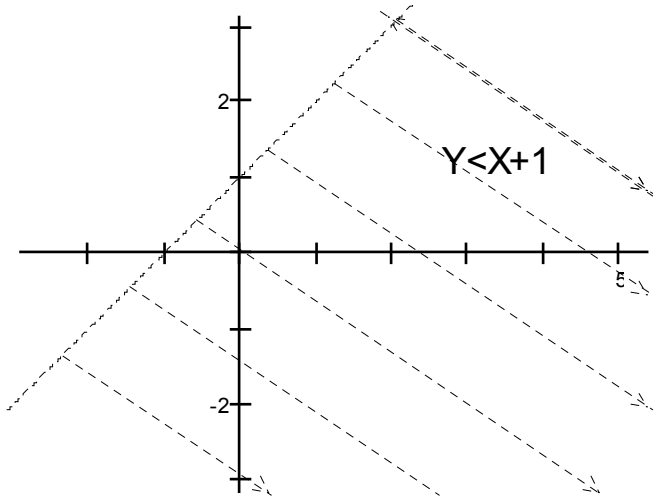


The line will always divide the plane into two areas, one in which points are a solution, and one where they are not.

Unless the line goes through the origin, you can most easily check with the point (0,0).

0<0+1 True!

So shade in the part of the plane that includes the origin. If the line goes through the origin, pick another point that is no on the line.



The process is the same if you have a horizontal or vertical line.

$y \leq 3$